To Accrete or not to Accrete: The Dilemma of the Recycling Scenario

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Abstract. We study the evolution of a low-mass X-ray binary by coupling a binary stellar evolution code with a general relativistic code that describes the behaviour of the neutron star. We find that non-conservative mass transfer scenarios are required to prevent the formation of submillisecond pulsars and/or the collapse to a black hole. We discuss the sweeping effects of an active magnetodipole rotator on the transferred matter as a promising mechanism to obtain highly non-conservative evolutions.

1. Introduction

1.1. Review of the Recycling Scenario

The “classical” scenario for the formation of millisecond radio pulsar binaries with low-mass companions envisages four main stages (see, e.g., Bhattacharya & van den Heuvel 1991 for a review):

(i) The magnetic moment $\mu$ of the newly formed radio pulsar ($\mu = BR^3$, where $B$ and $R$ are the surface magnetic field and the NS radius$^1$, respectively) decays spontaneously on an e-folding timescale $t_\mu \sim 10^7 - 10^8$ yr (e.g., Lyne, Anderson, & Salter 1982) from an initial value $\sim 10^{31}$ G cm$^3$ to a final value $\sim 10^{26} - 10^{27}$ G cm$^3$, when the decay probably stops (e.g., Bhattacharya & Srinivasan 1986).

(ii) Simultaneously, the pulsar spin period $P$ increases under magnetic dipole emission according to $L_{\text{PSR}} = (2/3c^3)\mu^2(2\pi/P)^4$ (where $L_{\text{PSR}}$ is the bolometric magneto-dipole luminosity and $c$ is the speed of light). The pulsar switches off when it eventually crosses the “death line” in the $\mu - P$ plane, i.e., the line defined by the relation $\mu_{26}/P^2 = 2 \times 10^{-5}$ (where $\mu_{26} = \mu/10^{26}$ G cm$^3$) below

$^1$Actually, since the NS is a relativistic object, $R$ is the NS circumferential equatorial radius, i.e., the proper circumference in the equatorial plane divided by $2\pi$. 

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which it is believed that the radio pulsar phenomenon does not take place (see, e.g., Ruderman & Sutherland 1975).

(iii) The companion star overflows its Roche lobe and transfers mass with angular momentum to the NS via a Keplerian accretion disc, thereby spinning it up to millisecond periods (close to the Keplerian period at inner rim of the accretion disk, see below) and back across the death line (recycling). During this phase the system is visible as a low-mass X-ray binary (LMXB).

(iv) Mass transfer ceases; the NS is again visible as a radio pulsar whose spin rate decays under magnetic dipole emission very slowly as $L_{\text{PSR}} \propto \mu^2$ and $\mu$ is reduced by three or four orders of magnitude. The end point is therefore a millisecond pulsar with a low-mass companion ($< 0.3 M_\odot$) that is the remnant of the $\sim 1 M_\odot$ donor.

1.2. Recycling in Transient Systems

Most LMXBs are NS Soft X-ray Transients (NSXT), i.e., transient systems harboring a NS (see Campana et al. 1998 for a review). Adopting the same conversion efficiency of the accreting matter energy into X-rays during the outbursts and the quiescent states (but see Barret et al. 2000 for a different explanation), the inferred variations in the accretion rate are by a factor $\sim 10^5$.

NSXTs can provide a direct evidence of the recycling scenario, since during stage (iii) the mass transfer rate varies up to five orders of magnitude. During the LMXB phase the accretion disk is truncated because of one of the following reasons: (i) the interaction with the magnetic field of the NS, which truncates the disc at the magnetospheric radius $R_M$, at which the accretion flow is channeled along the magnetic field lines towards the magnetic poles onto the NS surface; (ii) the presence of the NS surface itself at $R$; and (iii) the lack of closed Keplerian orbits for radii smaller than the marginally stable orbit radius, $R_{\text{MSO}}$ (at a few Schwarzschild radii from the NS centre, depending on mass and spin). The position of $R_M$ is determined by the instantaneous balance of the pressure exerted by the accretion disc and the pressure exerted by the NS magnetic field:

$$R_M = 1.0 \times 10^6 \phi \mu_26^{4/7} m_2^{-1/7} R_6^{-2/7} \dot{m}_2^{-2/7} \text{ cm},$$

where $\phi \leq 1$, $m$ is the NS gravitational mass in $M_\odot$, $R_6$ is the NS radius in units of $10^6$ cm, and $\dot{m}$ is the baryonic mass accretion rate$^2$ in Eddington units (the Eddington accretion rate is $1.5 \times 10^{-8} R_6 \ M_\odot \text{ yr}^{-1}$). Equation 1 shows that as $\dot{m}$ decreases, $R_M$ expands.

Accretion onto a spinning magnetized NS is centrifugally inhibited once $R_M$ expands beyond the corotation radius $R_{\text{CO}}$, at which the Keplerian angular frequency of the orbiting matter equals the NS spin: $R_{\text{CO}} = 1.5 \times 10^6 m^{1/3} P_{-3}^{2/3} \text{ cm}$ where $P_{-3}$ is the NS spin period in milliseconds. In this case the accreting matter could in principle be ejected from the system: this is called propeller phase.

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$^2$Actually, since the NS is a relativistic object, it is important to distinguish between the baryonic mass, roughly speaking a measure of the amount of matter, and the gravitational mass that is smaller by $\sim 3/5 GM^2/R \sim 0.1 M_c^2$, corresponding to the binding energy of the NS.
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(IIlarionov & Sunyaev 1975). Finally, if \( R_M \) further expands beyond the light-cylinder radius (where an object corotating with the NS attains the speed of light, \( R_{LC} = 4.8 \times 10^6 P^{-3} \) cm), the NS becomes generator of magnetodipole radiation and relativistic particles. Indeed, a common requirement of all the models for the emission from a rotating magnetic dipole is that the space surrounding the NS is free of matter up to \( R_{LC} \).

Let us consider the behaviour of a NSXT at the end of an outburst. Adopting \( \dot{m} \sim 1 \) in outburst, equation 1 gives \( R_M \sim R \sim 10^6 \) cm. In quiescence \( \dot{m} \sim 10^{-5} \) and \( R_M = (10^{-5})^{-2/3} \times 10^6 = 2.7 \times 10^7 \) cm \( > R_{LC} \), for spin periods up to a few milliseconds. Therefore, it is likely that, during the quiescent phase, a magneto-dipole emitter switches on (see, e.g., Stella et al. 1994; Burderi et al. 2001). In this case, as the NS \( \mu \) and \( P \) place such a system above the “death line”, it is plausible to expect that the NS turns-on as a millisecond radio pulsar until a new outburst episode pushes \( R_M \) back, close to the NS surface, quenching radio emission and restoring accretion.

2. To Accrete: Conservative Mass Transfer

A rapidly rotating NS can support a maximum mass (against gravitational collapse) much higher than the non-rotating mass limit, since the centrifugal force attenuates the effects of the gravitational pull. Conversely, if a rotating NS has a mass that exceeds the non-rotating limit (supramassive NS), it will be subject to gravitational collapse if it loses enough angular momentum, \( J \). In contrast to the standard behaviour, supramassive NSs spin-up just before collapse, even if they lose energy (Cook, Shapiro & Teukolsky 1992). The value of the maximum rotating and non-rotating mass depends on the equation of state (EOS) governing the NS matter, and the minimum allowed period for a given mass and EOS occurs when gravity is balanced by centrifugal forces at the NS equator (mass shedding limit). In this case the NS has the maximum \( J \) allowed for that given mass, \( J_{\text{max}} \). The NS radius, which is always in the order of \( 10^6 \) cm, depends on the mass, EOS, and \( J \) of the NS: a rapidly rotating NS can have a much larger radius than a non-rotating one (up to 40%, Cook, Shapiro & Teukolsky 1994).

In order to study the evolution of a LMXB in detail we coupled a modified version of the \texttt{rns} (rotating NS) public-domain code by Stergioulas & Morsink (1999), that allowed us to build a complete grid of relativistic equilibrium configurations, with the binary evolution evolution code \texttt{ATON} (D’Antona, Mazzitelli & Ritter, 1989). We assume the NS to be low-magnetized \( (B \sim 10^8 \) G). Our computations show that during the binary evolution, the companion transfers as much as \( 1 M_\odot \) to the NS, with an accretion rate of \( \sim 10^{-9} M_\odot \) yr\(^{-1} \). This rate is sufficient to keep the inner radius of the accretion disc, \( r_D \), in contact with the NS surface, thus preventing the onset of a propeller phase capable of ejecting a significant fraction of the matter transferred by the companion.

Matter leaves the inner radius of the disk \( r_D \) with a specific angular momentum \( j \) that is a function of \( r_D, M_G, \) and \( J \) (Bardeen 1970). We can therefore write
the evolutionary equations:

\[
\begin{align*}
\frac{dM_B}{dt} &= \dot{M}_C \\
\frac{dJ}{dt} &= f(J) \cdot \dot{M}_B 
\end{align*}
\]  

(2)

where $M_B$ is the baryonic mass of the star and $\dot{M}_C$ is the mass transfer rate from the companion, and $f(J) = j$ for $J < J_{\text{max}}(M_B)$ and $f(J) = dJ_{\text{max}}/dM_B$ for $J = J_{\text{max}}(M_B)$, since we have assumed that at mass shedding the matter of the disc dissipates angular momentum in order to accrete onto the NS, keeping the NS at the mass shedding limit.

Once the accretion ends, $R_M$ expands beyond $R_{\text{LC}}$ and a magneto-dipole rotator (radio pulsar) switches-on. In this situation is convenient to describe the NS evolution in terms of baryonic mass and total energy rather than in terms of baryonic mass and angular momentum:

\[
\begin{align*}
\frac{dM_B}{dt} &= 0 \\
\frac{dM_G}{dt} &= -(2/3c^5)\mu^2(2\pi/P)^4
\end{align*}
\]  

(3)

We integrate the differential equations 2 or 3 using a finite-differences method (see Lavagetto et al. 2004 for details).

In Figure 1 (left panel) we plot, in the mass-radius plane, the evolution of a system with a donor star of $1.15 M_\odot$, which loses $0.91 M_\odot$ during the accretion phase, which is representative of most of the simulations. To describe the NS we adopted the quite standard EOS FPS. The upper dashed curve is the mass-shedding limit, the lower dashed curve is the the sequence of stable non-rotating configurations, the thin solid line on the lower right is the limit beyond which the NS collapses onto a black hole, the thick solid line shows the evolution of the NS during the accretion phase, and, finally, the thick dashed line shows the
subsequent radio-pulsar phase. The first phase of accretion is characterized by a rapid spin up that brings the star to the mass-shedding limit, along which it evolves with $P$ slowly decreasing from 0.78 to 0.58 ms. Once the accretion stops during the radio-pulsar phase, $P$ decreases (because the NS is supramassive) to 0.54 ms, until the NS eventually collapses to a black hole.

All our simulations indicate that: (i) for most EOS, accretion-induced collapse to a black hole is almost unavoidable; (ii) in the absence of an external mechanism that brakes down the NS (such as gravitational radiation), a small fraction of the accreted matter ($\sim 0.1 M_\odot$) is sufficient to spin-up the NS to periods below one millisecond. In contrast, no submillisecond pulsar has been detected so far: the shortest observed spin period is 1.5 ms (Backer et al. 1982), uncomfortably higher than the theoretical predictions. Both these predictions are in contrast with the existence of the population of MSP all spinning at frequencies well above 1 ms.

3. Not to Accrete: Non-Conservative Mass Transfer

A non-conservative mass transfer has often been invoked to overcome the difficulties outlined above. Indeed the ejection of more than 90% of the transferred mass could prevent the gravitational collapse and keep the NS spin period above 1 ms. The propeller effect discussed in Sec. 1 is, in principle, a promising mechanism to obtain highly non-conservative evolutions. However, the virial theorem sets stringent limits on the fraction of matter that can be ejected in this phase. In fact, it states that, at any radius in the disk, the virialized matter has already liberated (via electromagnetic radiation) half of its available energy. Considering that $r_D \sim R$ for these kind of systems, the accreting matter has already radiated 50% of the whole specific energy obtainable from accretion ($\epsilon_{\text{acc}} \sim G M_\odot / R$) before setting onto the NS surface. Therefore to eject this matter $\sim 0.5 \epsilon_{\text{acc}}$ must be given back to it. As the only source of energy is that stored in the NS as rotational energy by the accretion process itself, the typical ejection efficiency is $\leq 50\%$, considering that once the system has reached the spin equilibrium, no further spin-up takes place and the storage of accretion energy in rotational energy is impossible. Thus, the accreted mass is $\geq 0.5 M_\odot$, certainly enough to spin-up the NS to submillisecond periods. An alternative viable hypothesis to explain the lack of ultrafast rotating NSs is that gravitational wave emission balances the torque due to accretion (Ushomirsky, Bildsten, & Cutler 2000), although this effect cannot prevent the gravitational collapse.

The only way to overcome these difficulties is to obtain ejection efficiencies close to unity. This is indeed possible if the matter is ejected so far away from the NS surface ($r \gg R$) that it has an almost negligible binding energy $\epsilon = G M_\odot / r \ll \epsilon_{\text{acc}}$. As the NS is spinning very fast, the switch-on of a radio pulsar is unavoidable once $R_M > R_{LC}$. In this case the pressure exerted by the radiation field of the radio pulsar may overcome the pressure of the accretion disk, thus determining the ejection of matter from the system. Once the disk has been swept away, the radiation pressure stops the infalling matter as it overflows the inner Lagrangian point $L_1$, where $\epsilon \simeq 0$. 
The push on the accretion flow exerted by the magnetic field of the NS can be described in terms of an outward pressure (we use the expressions outward or inward pressure to indicate the direction of the force with respect to the radial direction): \( P_{\text{MAG}} = \frac{B^2}{4\pi} = 7.96 \times 10^{14}\mu_{26}^{-10} \frac{r_0^6}{r_6^2} \text{ dy/cm}^2 \), where \( r_0 \) is the distance from the NS center in units of 10\(^6\) cm. If the disk terminates outside \( R_{\text{LC}} \), the outward pressure is the radiation pressure of the rotating magnetic dipole, which, assuming isotropic emission, is \( P_{\text{PSR}} = 2.04 \times 10^{32} P_{-3}^{-4} \mu_{26}^{-10} \frac{r_0^6}{r_6^2} \text{ dy/cm}^2 \). In Figure 1 the two outward pressures (\( P_{\text{MAG}} \) and \( P_{\text{PSR}} \)) are shown as bold lines for typical values of the parameters (see figure caption).

The flow, in turn, exerts an inward pressure on the field. For a Shakura–Sunyaev accretion disc (see, e.g., Frank, King & Raine 1992): \( P_{\text{DISK}} = 1.02 \times 10^{16} \alpha^{-9/10}n_{0.615}^{-1}L_{37}^{17/20}m_{140}^{-1/40}R_6^{17/20}f^{17/5}r_6^{-21/8} \text{ dy/cm}^2 \), where \( \alpha \) is the Shakura–Sunyaev viscosity parameter, \( n_{0.615} = n/0.615 \sim 1 \) for a gas with solar abundances (where \( n \) is the mean particle mass in units of the proton mass), and \( f = [1 - (R_6/r_0)^{1/2}]^{-1/4} \leq 1 \). We measure \( \dot{M} \) in units of \( L_{37} = L/10^{37} \text{ ergs/s} \) from \( L = GM\dot{M}/R \).

In Figure 1 (right panel) the inward disc pressure for a luminosity \( L_{\text{MAX}} \), corresponding to the outburst luminosity of an NSXT, is shown as a thin solid line. The disc pressure line, which intersects \( P_{\text{MAG}} \) at \( R_{\text{LC}} \), defines a critical luminosity \( L_{\text{switch}} \), shown as a dashed line in Fig. 1, at which the radio pulsar switches-on.

The intersections of the \( P_{\text{DISK}} \) line corresponding to \( L_{\text{MAX}} \) with each of the outward pressure lines define equilibrium points between the inward and outward pressures. The equilibrium is stable at \( r = R_m \), and unstable at \( r = R_{\text{STOP}} \), which can be derived equating \( P_{\text{PSR}} \) and \( P_{\text{DISK}} \): \( R_{\text{STOP}} \sim 8 \times 10^{11} \alpha^{-36/25}n_{0.615}^{-8/5}R_6^{34/25}f^{136/25}L_{37}^{34/25}m_{140}^{-16/5}\mu_{26}^{1/5}P_{-3}^{25/5} \text{ cm} \). In fact, as \( P_{\text{MAG}} \) is steeper than \( P_{\text{DISK}} \), if a small fluctuation forces the inner rim of the disc inward (outward), in a region where the magnetic pressure is greater (smaller) than the disc pressure, this results in a net force that pushes the disc back to its original location \( R_m \). As \( P_{\text{PSR}} \) is flatter than \( P_{\text{DISK}} \), with the same argument is easy to see that no stable equilibrium is possible at \( R_{\text{STOP}} \) and the disc is swept away by the radiation pressure. This means that, for \( r > R_{\text{STOP}} \), no disc can exist for any luminosity \( \leq L_{\text{MAX}} \). The sudden drop in the mass-transfer rate during the quiescent phase of a NSXT initiates a phase that we termed “radio ejection”, in which the mechanism that drives mass overflow through \( L_1 \) is still active, while the pulsar radiation pressure prevents mass accretion. As the matter released from the companion cannot accrete, it is now ejected as soon as it enters the Roche lobe of the primary. The distance of \( L_1 \) from the NS center, \( R_{\text{L1}} \), depends only on the orbital parameters. In the approximation given by Paczynski (1971), we can impose \( R_{\text{STOP}}/R_{\text{L1}} = 1 \) and solve for the orbital period:

\[
P_{\text{crit}} = 1.05 \times (\alpha^{-36}n_{0.615}^{-40}R_6^{34/35}L_{36}^{51/25}m_{140}^{-10}\mu_{26}^{-24/5}P_{-3}^{48/5}g(m, m_2) \text{ h} \quad (4)
\]

where \( g(m, m_2) = [1 - 0.462(m_2/(m + m_2))^{1/3}]^{-3/2}(m + m_2)^{-1/2} \), and \( m_2 \) is the mass of the companion in solar masses. For a system with \( P_{\text{orb}} > P_{\text{crit}} \) (e.g., \( P_{\text{orb}} = 3 \text{ hr} \) in Fig. 1), once a drop of the mass-transfer rate has started the radio ejection, a subsequent restoration of the original mass-transfer rate is unable to
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quench the ejection process (see Burderi et al. 2001 for a detailed discussion of this effect), thus prolonging the ejection phase.

This “radio-ejection” phase envisaged by Burderi et al. (2001), has been spectacularly demonstrated by the discovery of PSR J1740-5340, an eclipsing MSP, with a spin period of 3.65 ms and an orbital period of 32.5 h (D’Amico et al. 2001), very close to $P_{\text{crit}}$ for the parameters of this system (Burderi, D’Antona, & Burgay 2002). The peculiarity of this system is that the companion is still overflowing its Roche lobe. This is demonstrated by the presence of matter around the system that causes the long lasting and sometimes irregular radio eclipses, and by the shape of the optical light curve, which is well modeled assuming a Roche-lobe deformation of the mass losing component (Ferraro et al. 2001). An evolutionary scenario for this system has been proposed by Burderi et al. (2002), who provided convincing evidence that PSR J1740-5340 is an example of a system in the radio-ejection phase, by modeling the evolution of the possible binary system progenitor.

In conclusion, while the evolution without a radio-ejection phase implies that a large fraction of the transferred mass is accreted onto the NS (because of the constraints imposed by the virial theorem), we have demonstrated that the switch-on of a radio pulsar (associated to a significant drop in mass transfer) could determine long episodes characterized by ejection efficiencies close to 100%, as the matter is ejected before it falls into the deep gravitational potential well of the primary. This phenomenon can prevent gravitational collapse and ultra-fast spinning NSs.

References

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