Introduction
Possibly as many as half of the bright X-ray sources in the galactic globular clusters are binaries with ultrashort orbital periods (≤10 min). Two of the five periods known are 11.4 min (in NGC 6624) and 20.6 min (in NGC 6712). The 11.4 min system has a negative period derivative. This high fraction of ultrashort periods is in marked contrast to the case of bright X-ray sources in the galactic disk, where such periods are less common.

One of the scenarios to explain the ultrashort periods starts from a binary with a neutron star (NS) and a main-sequence star. For a small range of initial orbital periods, strong magnetic braking can shrink the orbit sufficiently that the system evolves to an ultrashort minimum period. This way an orbital period shorter than 11 min can be reached [2]. At 11 min, the period derivative may be negative or positive, depending on whether the system evolves to the period minimum, or has already rebounded. We investigate which initial systems can reach ultrashort periods within the age of the globular clusters (10-13 Gyr) and what the probability is to observe these systems as X-ray sources.

The evolution code
We calculate different sets of models using the STARS binary stellar evolution code, developed by Eggleton [1], but with updated physics. The primary (NS) is treated as a 1.4 M⊙ point mass. Sources of angular momentum loss are gravitational waves, partially conservative mass transfer, and magnetic braking (MB). We use two different MB prescriptions:

\[ \frac{dJ}{dt} = -3.8 \times 10^{-39} \eta M_2 R_2^2 \omega^3 \text{dyn cm}, \quad \text{(MB1)} \]

from [3] but with \( \eta \) to regulate the strength, and

\[ \frac{dJ}{dt} = -4.6 \times 10^{-39} \left( \frac{R_2}{M_2} \right)^{0.5} \omega^2 \text{dyn cm}, \quad \text{(MB2)} \]

from [4]. Here \( M_2, R_2, \) and \( \omega \) are the mass, radius and spin velocity of the secondary. In the second law, \( \omega_2 = \omega_1 \) for \( \omega \leq \omega_{\text{crit}} \) and \( \omega_2 = \omega_{\text{crit}}^2 \) for \( \omega > \omega_{\text{crit}} \), where \( \omega_{\text{crit}} \) depends on the envelope convection timescale. Thus, in this prescription the magnetic-braking strength saturates for rapidly rotating stars. Because tidal effects keep the spin of the star synchronised to the orbit, the magnetic braking effectively removes angular momentum from the orbit. All models that we present here have \( \omega = 0.01 \), the metallicity of NGC 6624.

The calculated models for MB1
We calculated a grid of models for MB1 with \( \eta = 1 \), initial masses between 0.7 and 1.5 M⊙, and initial periods between 0.35 and 3 d. The grid is refined in period around the bifurcation period (\( \omega_{\text{crit}} \)) between converging and diverging systems. We consider models that converge after the Hubble time as diverging.

Figure 1 shows that orbits with low initial period converge to about 70 min, and orbits with high orbital period diverge to several days. A small range in between leads to ultrashort systems. It is clear that since only systems with initial periods from a narrow range of values evolve to such short period minima, these are very rare in nature.

Statistics for MB1
To determine the probability of observing an ultracompact binary produced this way, we perform statistics on the t-P tracks. We choose a random initial period from a flat distribution in log \( P \) and determine its t-P track by interpolation. Once the track is known, we choose a random moment in time between 10 and 13 Gyr (the dotted lines in Fig. 1) and determine the orbital period at that moment. If the system has passed its period minimum, or has no mass transfer at that moment, we reject it. Of \( 10^6 \) random systems with \( M = 1.1 \) M⊙ produced, about 10% is accepted. These are shown in Fig. 2a. Figure 2b shows the corresponding period distribution.

Conclusions
We find that the strong magnetic braking of MB1 with \( \eta = 1 \) can lead to ultracompact X-ray binaries within a Hubble time. To create a binary that has its minimum period in the ultrashort-period regime, one has to carefully select an initial period just under the bifurcation period. The systems that reach an ultrashort period remain there only for a short time, as can be seen from the steep tracks in Fig. 1. These factors combined make it unlikely that such a system can be observed. The distribution also suggests that for each binary with \( \omega_{\text{crit}} \leq 11 \) min, there are about 100 binaries with \( \omega_{\text{crit}} \geq 20 \) min and several thousand with \( \omega_{\text{crit}} \leq 50 \) min.

If creating an 11 min binary is unlikely with MB1 & \( \eta = 1 \), it becomes impossible for the perhaps more realistic lower \( \eta \) or MB2. An alternative scenario to create ultracompact binaries is a spiral-in, possibly initiated by a stellar collision in the dense globular clusters, or tidal capture. Since these scenarios only allow positive period derivatives, it may be worthwhile to reconsider the gravitational acceleration of the 11 min system in NGC 6624.

References

Other MB prescriptions
Figure 3 compares t-P tracks of a selection of models with different MB prescriptions and strengths. The figure shows that the tracks with weaker MB converge slower, as can be expected. The models with MB2 seem to lie between MB1, \( \eta = 0.25 \) and MB1, \( \eta = 0 \) in this case. We performed the same statistics as for MB1, \( \eta = 1 \). The period distributions for each mass have been added using a flat mass distribution. We compare the results in Fig. 4. Figure 4 shows that the four distributions mainly differ in the low-period cut-off. Despite low-number statistics in the ultracompact regime, the distribution suggests that among 10^3 binaries with MB1, \( \eta = 1 \) and 0.5 ≤ \( P_1 \) ≤ 2.5 d, there is 1 X-ray binary with a period \( \omega_{\text{crit}} \geq 11.4 \) min and ~ 100 systems with \( \omega_{\text{crit}} \leq 20.6 \) min. The period cut-off for MB1, \( \eta = 0.25 \) lies around 23 min, whereas the models with \( \eta = 0 \) or MB2 both have cut-offs at 70 min.